Propiedades de los logaritmos.

$$x = \log_a b \quad \Rightarrow \quad a^x = b$$

$$\log_a 1 = 0$$

$$\log_a a = 1$$

$$\log_a a^n = n$$

$$\log_a (x \cdot y) = \log_a x + \log_a y$$

$$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$$

$$\log_a (x^n) = n \log_a x$$

$$\log_a \left(\sqrt[n]{x} \right) = \frac{1}{n} \log_a x$$

Resolver las ecuaciones logarítmicas

$$\log 2 + \log\left(11 - x^2\right) = 2\log(5 - x)$$

$$\log\left[2\left(11 - x^2\right)\right] = \log(5 - x)^2$$

$$2\left(11 - x^2\right) = (5 - x)^2$$

$$3x^2 - 10x + 3 = 0$$

$$x = 3 \qquad 11 - 3^2 > 0 \qquad 5 - 3 > 0$$

$$x = \frac{1}{3} \qquad 11 - \left(\frac{1}{3}\right)^2 > 0 \qquad 5 - \frac{1}{3} > 0$$

$$2\log x = 3 + \log \frac{x}{10}$$

$$2\log x = 3 + \log x - \log 10$$

$$\log x = 3 - 1 \qquad \log x = 2 \qquad x = 100$$

$$\log x + \log(x + 3) = 2\log(x + 1)$$

$$\log[x(x + 3)] = \log(x + 1)^2$$

$$x(x + 3) = (x + 1)^2$$

$$x^2 + 3x = x^2 + 2x + 1 \qquad\qquad x = 1$$

$$\frac{\log(16 - x^2)}{\log(3x - 4)} = 2$$

$$\log(16 - x^2) = 2\log(3x - 4)$$

$$\log(16 - x^2) = \log(3x - 4)^2 \qquad\qquad (16 - x^2) = (3x - 4)^2$$

$$10x^2 - 24x = 0 \qquad x = 0 \qquad x = \frac{24}{10} = \frac{12}{5}$$

$$4\log\left(\frac{x}{5}\right) + \log\left(\frac{625}{4}\right) = 2\log x$$

$$\log\left(\frac{x}{5}\right)^4 + \log\left(\frac{625}{4}\right) = \log x^2 \qquad\qquad \log\left(\frac{x^4}{625} \cdot \frac{625}{4}\right) = \log x^2$$

$$\log\left(\frac{x^4}{4}\right) = \log x^2 \qquad\qquad \frac{x^4}{4} = x^2 \qquad\qquad x^4 - 4x^2 = 0$$

$$x = 0 \qquad x = -2 \qquad x = 2$$

$$2\log x - 2\log(x + 1) = 0$$

$$\log x^2 - \log(x + 1)^2 = \log 1$$

$$\log\frac{x^2}{(x + 1)^2} = \log 1 \qquad\qquad \frac{x^2}{(x + 1)^2} = 1$$

$$2x + 1 = 0 \qquad\qquad x = -\frac{1}{2} \qquad\qquad \text{Sin solución}$$

$$\log x = \frac{2 - \log x}{\log x}$$

$$(\log x)^2 + \log x - 2 = 0 \qquad \log x = t$$

$$t^2 + t - 2 = 0 \qquad t = 1 \qquad t = -2$$

$$\log x = 1 \qquad x = 10 \qquad \log x = -2 \qquad x = 10^{-2} = \frac{1}{100}$$

$$\log(25 - x^3) - 3\log(4 - x) = 0$$

$$\log(25 - x^3) = \log(4 - x)^3 \qquad (25 - x^3) = (4 - x)^3$$

$$25 - x^3 = 64 - 48x + 12x^2 - x^3$$

$$12x^2 - 48x + 39 = 0 \qquad x = 2 \pm \frac{\sqrt{3}}{2}$$

$$\frac{\log(35 - x^3)}{\log(5 - x)} = 3$$

$$\log(35 - x^3) = 3\log(5 - x)$$

$$\log(35 - x^3) = \log(5 - x)^3 \qquad (35 - x^3) = (5 - x)^3$$

$$x^2 - 5x + 6 = 0 \qquad x = 2 \qquad x = 3$$

$$\log_5 x + \frac{\log_5 125}{\log_5 x} = \frac{7}{2}$$

$$(\log_5 x)^2 - \frac{7}{2}\log_5 x + \log_5 125 = 0$$

$$2(\log_5 x)^2 - 7\log_5 x + 6 = 0 \qquad \log_5 x = t$$

$$2t^2 - 7t + 6 = 0 \qquad t = 2 \qquad t = \frac{3}{2}$$

$$\log_5 x = 2 \qquad x = 25 \qquad \log_5 x = \frac{3}{2} \qquad x = \sqrt{5^3} = 5\sqrt{5}$$