

$$y = \frac{1+|x|}{1-|x|}$$

$$\begin{cases} \frac{1+x}{1-x} & x \geq 0 \\ \frac{1-x}{1+x} & x < 0 \end{cases}$$

$$\frac{\frac{1-x}{1+x}}{0} \quad \frac{\frac{1+x}{1-x}}{0}$$

Ptas Corte

$$y = \frac{1-x}{1+x} \quad \text{PC} \begin{cases} x=0 \rightarrow y=1 & (0,1) \\ y=0 \rightarrow x=1 & (1,0) \end{cases} \text{ fuera de su definición}$$

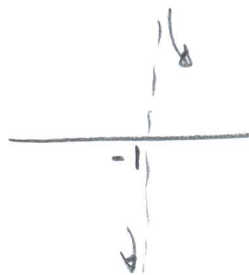
ASÍMPTOTAS

Verticales

$$\boxed{x = -1}$$

$$\lim_{x \rightarrow -1^+} \frac{1-x}{1+x} = \frac{+}{+} = +\infty$$

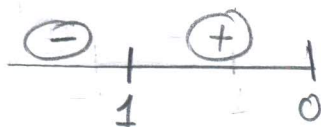
$$\lim_{x \rightarrow -1^-} \frac{1-x}{1+x} = \frac{+}{-} = -\infty$$



Horizontales

$$\lim_{x \rightarrow \infty} \frac{1-x}{1+x} = \frac{\infty}{\infty} \stackrel{L}{=} -1 \quad \boxed{y = -1}$$

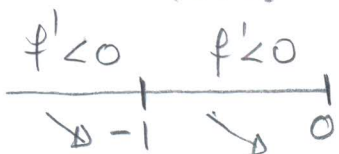
SIGNO



CRECIMIENTO

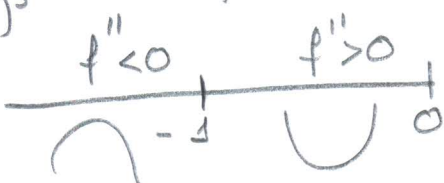
$$f'(x) = \frac{-(1+x) - (1-x)}{(1+x)^2} = \frac{-2}{(1+x)^2}$$

$$f'(x) = 0 \rightarrow \nexists \text{ PC}$$



$$f''(x) = \frac{4}{(1+x)^3}$$

$$f''(x) = 0 \rightarrow \nexists \text{ P.I}$$



$$y = \frac{1+x}{1-x}$$

P.C.  $\left\{ \begin{array}{l} x=0 \rightarrow y = -1 \quad (0, -1) \\ y=0 \rightarrow 1+x=0 \rightarrow x=-1 \quad (-1, 0) \end{array} \right.$

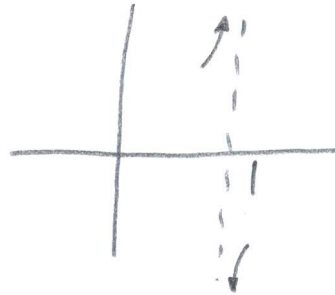
### ASINTOTAS

#### Verticales

$$1-x=0 \rightarrow \boxed{x=1}$$

$$\lim_{x \rightarrow 1^+} \frac{1+x}{1-x} = -\infty$$

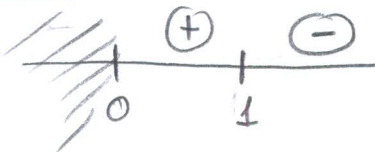
$$\lim_{x \rightarrow 1^-} \frac{1+x}{1-x} = +\infty$$



#### Horizontales

$$y = \lim_{x \rightarrow \infty} \frac{1+x}{1-x} = \frac{\infty}{\infty} \rightarrow \boxed{y=-1}$$

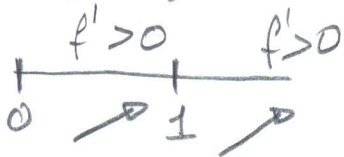
### SIGNO



### CRECIMIENTO

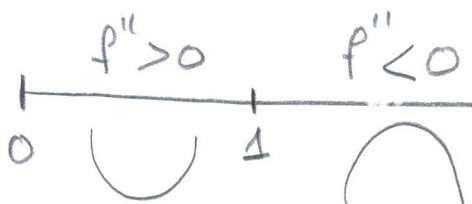
$$f'(x) = \frac{(1-x) + (1+x)}{(1-x)^2} = \frac{2}{(1-x)^2}$$

$$f'(x) = 0 \neq \text{P.C.}$$



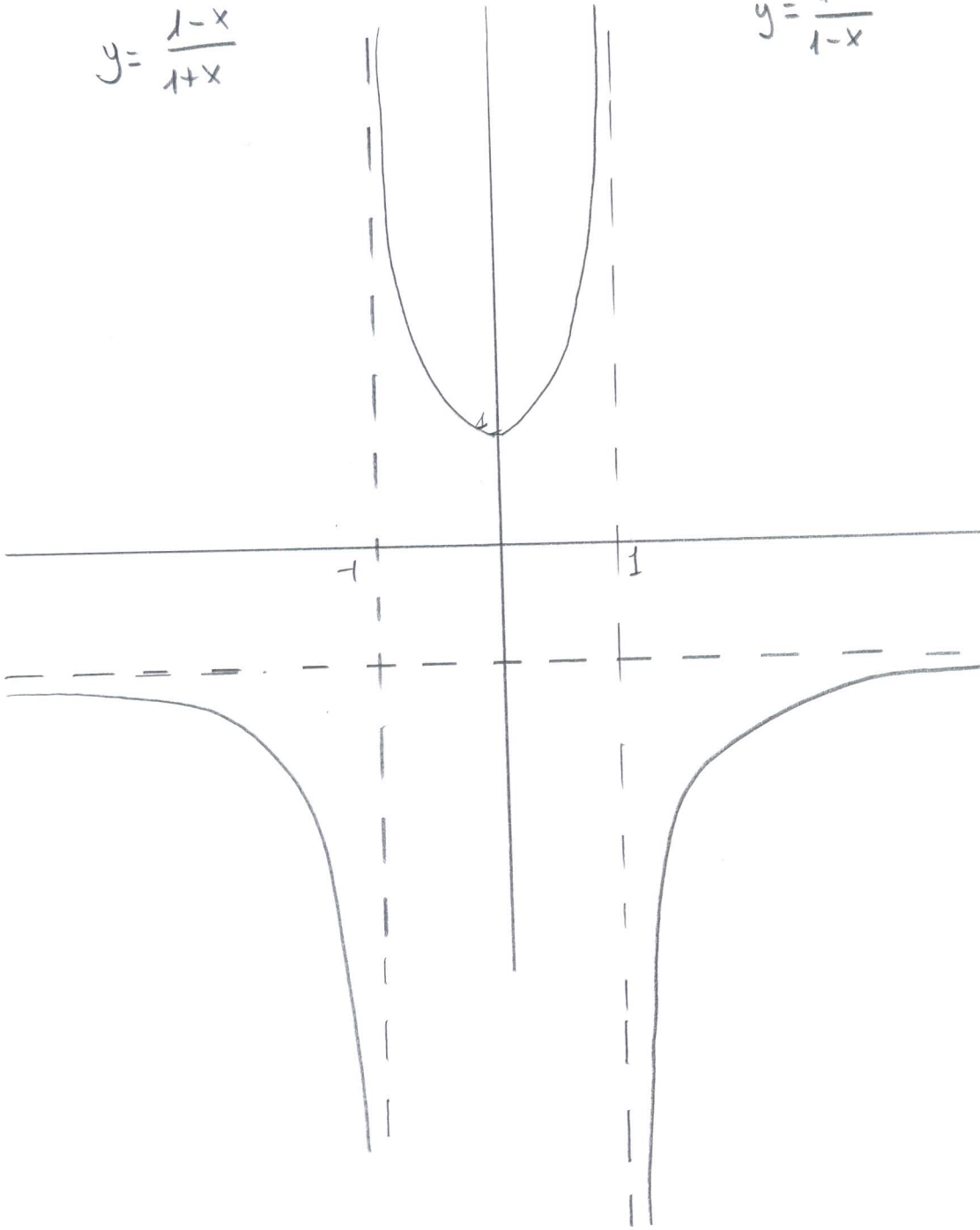
$$f''(x) = \frac{4}{(1-x)^3}$$

$$f''(x) = 0 \neq \text{P.I.}$$



$$y = \frac{1-x}{1+x}$$

$$y = \frac{1+x}{1-x}$$



$$f(x) = x - \sqrt{x^2 - 1}$$

DOMINIO

$$x^2 - 1 \geq 0$$



$$D = (-\infty, -1] \cup [1, +\infty)$$

PTOS CORTE

$$x=0 \rightarrow y \nexists$$

$$y=0 \rightarrow x - \sqrt{x^2 - 1} = 0 \rightarrow x = (\sqrt{x^2 - 1})^2 \rightarrow 0 = -1 \nexists$$

ASÍNTOTAS

• VERTICALES

$$\lim_{x \rightarrow 1} f(x) = 1$$

$$\lim_{x \rightarrow -1} f(x) = -1$$

• HORIZONTALES

$$\lim_{x \rightarrow +\infty} x - \sqrt{x^2 - 1} = \infty - \infty$$

$$\lim_{x \rightarrow +\infty} \frac{(x - \sqrt{x^2 - 1})(x + \sqrt{x^2 - 1})}{(x + \sqrt{x^2 - 1})} = \lim_{x \rightarrow +\infty} \frac{x^2 - x^2 + 1}{x + \sqrt{x^2 - 1}} =$$

$$= \lim_{x \rightarrow +\infty} \frac{1}{x + \sqrt{x^2 - 1}} = \frac{1}{+\infty} = \boxed{0}$$

$$\lim_{x \rightarrow -\infty} x - \sqrt{x^2 - 1} = -\infty - \infty = \boxed{-\infty} \nexists \text{ A.H.}$$

• OBLICUA

$$y = mx + n \quad \boxed{y = 2x}$$

$$m = \lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \lim_{x \rightarrow -\infty} \frac{x - \sqrt{x^2 - 1}}{x} = \frac{\infty}{\infty}$$

$$\lim_{x \rightarrow -\infty} \frac{-1 - \sqrt{1}}{-1} = 2$$

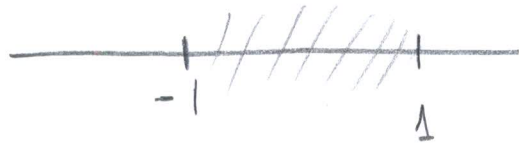
$$n = \lim_{x \rightarrow -\infty} x - \sqrt{x^2 - 1} - 2x = \lim_{x \rightarrow -\infty} (-x - \sqrt{x^2 - 1}) \stackrel{?}{=} 0$$

## CRECIMIENTO

$$f'(x) = 1 - (x^2-1)^{-1/2} \cdot 2x \cdot \frac{1}{2} = 1 - \frac{x}{\sqrt{x^2-1}} = \frac{\sqrt{x^2-1} - x}{\sqrt{x^2-1}}$$

$$f'(x) = 0 \rightarrow \sqrt{x^2-1} - x = 0 \rightarrow \sqrt{x^2-1} = x$$

$$-1 = 0 \rightarrow \nexists \text{ P. Críticos}$$

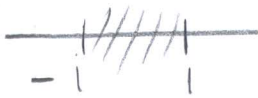


$$f'(-2) = \frac{+}{+} \quad (-\infty, -1] \text{ CREC.}$$

$$f'(2) = \frac{-}{+} \quad [1, +\infty) \text{ DCR.}$$

## CURVATURA

$$f'' = \frac{\left[ \frac{1}{2} (x^2-1)^{-1/2} \cdot 2x - 1 \right] \sqrt{x^2-1} - \left[ \sqrt{x^2-1} - x \right] \cdot \frac{1}{2} (x^2-1)^{-1/2} \cdot 2x}{x^2-1}$$

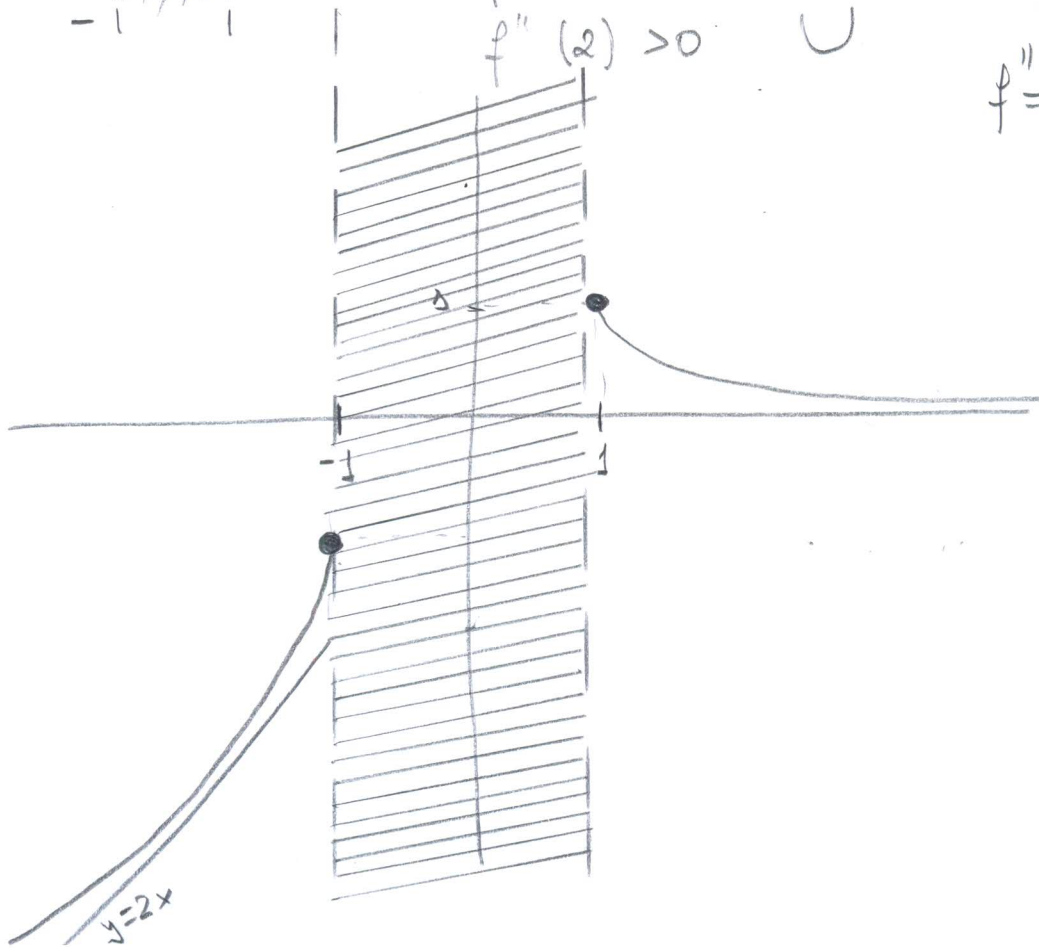


$$f''(-2) > 0 \quad \cup$$

$$f''(2) > 0 \quad \cup$$

$$f'' = \frac{-\sqrt{x^2-1} + x^2}{\sqrt{x^2-1} (x^2-1)}$$

$$f'' = 0 \rightarrow \nexists \text{ P. I.}$$



$$y = \left| \frac{x^2 - x}{x^2 - 1} \right| \quad y = \left| \frac{x}{x+1} \right| \quad \left\{ \begin{array}{ll} \frac{x}{x+1} & x < -1 \\ -\frac{x}{x+1} & -1 < x < 0 \\ \frac{x}{x+1} & x \geq 0 \end{array} \right.$$



PC  $\left\{ \begin{array}{l} x=0 - y=0 \quad (0,0) \\ y=0 - x=0 \quad (0,0) \end{array} \right.$

ASINTOTAS

Verticales  $x = -1$

$$\lim_{x \rightarrow -1^+} \frac{x}{x+1} = +\infty$$

$$\lim_{x \rightarrow -1^-} \frac{x}{x+1} = +\infty$$

Horizontales

$$y = \lim_{x \rightarrow \infty} \frac{x}{x+1} = \frac{\infty}{\infty} \rightarrow \boxed{y=1}$$

CRECIMIENTO

$$f'(x) = \frac{1}{(x+1)^2} \quad f'(x) = 0 \rightarrow \# \text{ P.C}$$



CURVATURA

$$f''(x) = \frac{-2}{(x+1)^3} \quad f''(x) = 0 \rightarrow \# \text{ P.I}$$

